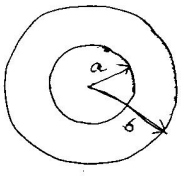
Physics III
ISI B.Math
Backpaper Exam : December 28 ,2015

Total Marks: 100
Time : 3 hours
Answer all questions.

1. (**Marks: (2 + 3 + 2 + 3 + 5 + 5**)



a) A hollow spherical shell carries charge density

$$\rho = \frac{k}{r^2}$$

in the region $a \leq r \leq b$. Find the electric field in the three regions (i)$r < a$,(ii) $a < r < b$, (iii) $r > b$. Plot $|\mathbf{E}|$ as a function of r.
b) A point charge q is held a distance d above an infinite grounded conducting plane.
i) Find the force of attraction between the charge and the plane
ii)Find the energy of the configuration

2. (Marks: 3 + 8 + 9)

(a) Why does a *sudden* unplugging of an electrical device like a toaster or an iron often result in drawing a spark?
(b) A battery of emf $\mathcal{E}$ is connected to a circuit of resistance R and inductance L. Find the current in the circuit as a function of time.
(c) Suppose we replace the resistor with a capacitor of capacitance C charged to a potential V and replace the battery by a switch. At time $t = 0$ the switch is closed. Find the current in the circuit as a function of time. How will your answer change if the resistor R is put back in series with C and L?

3. (Marks: 4 + 4 + 6 + 6)

(a) Write down the full set of Maxwell's equations in differential form.
(b) Derive the continuity equation : $\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0$ from Maxwell's equations, where the symbols have their usual meanings.
(c) Show that , for Maxwell's equations in vacuum, each Cartesian component of $\mathbf{E}$ and $\mathbf{B}$ satisfies the 3-D wave equation

$$\nabla^2 f = \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$$

with $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$
(d) Write down the real electric and magnetic fields corresponding to a monochromatic plane wave representing an electric field of amplitude E_0 , frequency ω and phase angle zero that is travelling

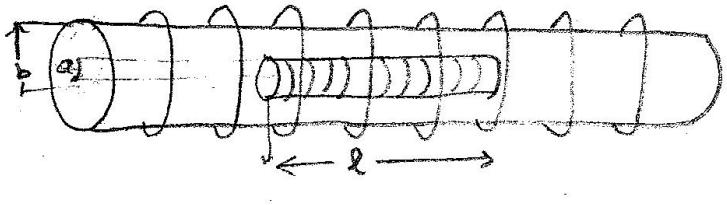
in the negative x direction and polarized in the z-direction. Find the energy density stored in the electromagnetic field corresponding to this solution.

4. (Marks: 6 + 7 + 7)

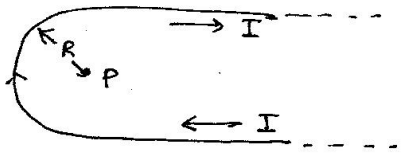
(a) Find the potential corresponding to the electric field given below. $\mathbf{E} = k[y^2\hat{\mathbf{x}}+(2xy+z^2)\hat{\mathbf{y}}+2yz\hat{\mathbf{z}}]$

(b) A beam of electrons with charge q and mass m is passed through crossed uniform electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$(mutually perpendicular,and both of them perpendicular to the beam). The electric field is then adjusted until the beam does not get deflected at all. What was the speed of the particles in terms of E and B?
(c) the electric field is then turned off and the radius of curvature R of the beam is measured, as deflected by the magnetic field alone. What is the charge-to-mass ratio $\frac{q}{m}$ of the particles in terms of E, B, and R ?

5. (Marks: 10 + 10)



(a) A short solenoid (length l and radius a, with n_1 turns per unit length) lies on the axis of a very long solenoid (radius b, n_2 turns per unit length)as shown in figure above Current I flows in the short solenoid. What is the flux through the long solenoid ? Find the mutual inductance of the system.



(b) (i) A long wire is bent into the hairpin like shape shown in the figure. Find an exact expression for the magnetic field at the point P which lies at the centre of the half-circle.